MA1161
Midterm Exam
Spring 2007

Name:________________________________

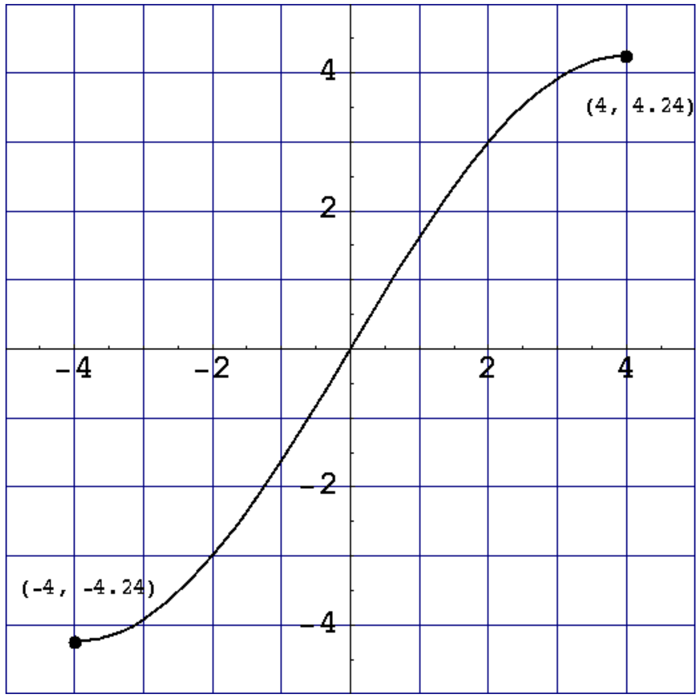
Calculator section: **You may use your calculator on this section. You must show all work in order to recieve full credit.** Once you have handed in this section you may not have it back.

Circle your instructor and section number.

Instructor	Section
Schaefer	R01
Xiao	R02
Butler	R03
Miller	R04
Garcia	R05
Zei	R06
Humes	R08
Cui	R10

Page	Score	
1		/ 6
2		/ 8
3		/ 6
4		/ 10
5		/ 10
6		/ 9
7		/ 10
8		/ 5
Calculator total		/ 64
9		/ 16
10		/ 14
11		/ 6
Non-Calculator total		/ 36
Final score		/ 100

1. The entire graph of a function, $y = f(x)$, is shown below and its endpoints are labeled.



(a) Does this function have an inverse? How can you tell? [1]

Solution.

Yes, it passes the horizontal line test. □

(b) What is the domain of $f(x)$? [-4,4] [1]

(c) What is the range of $f(x)$? [-4.24,4.24] [1]

(d) Give a value for x where $f(x)$ is increasing. Any value in (-4,4) [1]

(e) Give a value for x where $f(x)$ is concave down. Any value in (0,4) [1]

(f) What is $f^{-1}(3)$? $f^{-1}(3) = 2$ [1]

2. After the release of radioactive material into the atmosphere from a nuclear power plant at Chernobyl (Ukraine) in 1986, the hay in Austria was contaminated by iodine 131 which has a half-life of 8 days.

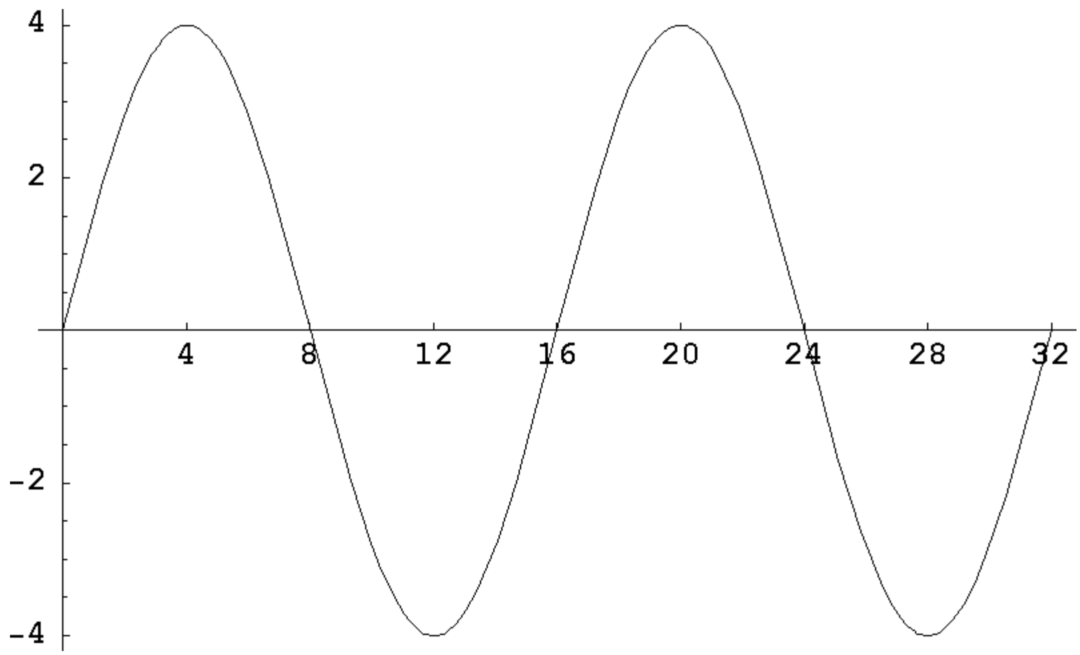
   (a) Find a formula of the form $P(t) = P_0 e^{kt}$ which gives the amount of iodine 131 left after t days.

   $P(t) = P_0 e^{(-\frac{1}{8}\ln 2)t}$ [4]

   (b) If it is all right right to feed the hay to cows when 10% of the iodine 131 remains, how long do the farmers have to wait to use the hay?

   $t \approx 26.6$ days [4]

3. Find a possible formula for the sinusoidal function $y = g(x)$ graphed below.



(a) What is the amplitude? $A = 4$ [1]

(b) What is the period? $P = 16$ [1]

(c) Give a formula for the function.

$g(x) = 4\sin(\frac{\pi}{8}x)$ [2]

(d) Find a value of x where $g'(x) = 0$. $x =$4, 12, 20, or 28 [2]

4. At a time t seconds after it is thrown up in the air, a tomato is at a height of $f(t) = -4.9t^2 + 25t + 3$ meters.

   (a) What is the average velocity of the tomato during the first 2 seconds? Give units.

   15.2 m/s [2]

   (b) Find the exact instantaneous velocity of the tomato at $t = 2$. Give units.

   5.4 m/s [3]

   (c) What is the acceleration of the tomato at $t = 2$? Give units.

   -9.8 m/s^2 [3]

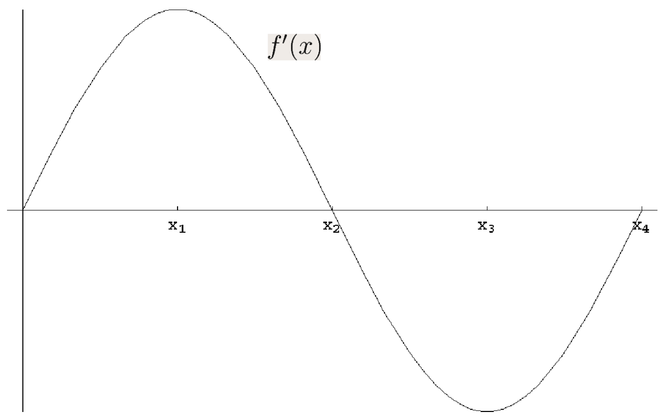
   (d) When does the tomato reach its highest point?

   2.55 s [1]

   (e) How long does the tomato stay in the air?

   5.1 s [1]

5. Consider the graph of the derivative of a function (not the function itself) given below:



(a) At which point(s) x_1, x_2, x_3, or x_4 is the function $f(x)$ (not $f'(x)$) increasing?

x_1 [2]

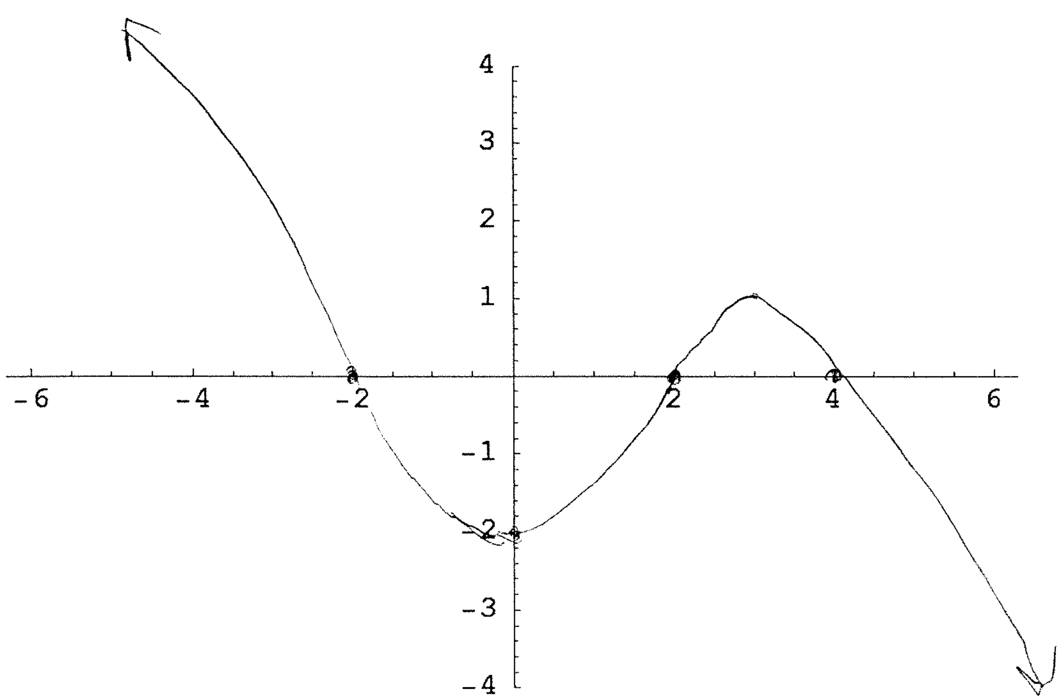
(b) At which point(s) x_1, x_2, x_3, or x_4 is the function $f(x)$ (not $f'(x)$) concave down?

x_2 [3]

6. Draw the graph of a function fulfilling the following properties:

- $f(x) > 0$ for $x < -2$ and $2 < x < 4$; $f(x) < 0$ otherwise.
- $f'(x) > 0$ for $0 < x < 3$; $f'(x) < 0$ otherwise.

[5]



7. The population of the world in billions of people can be modeled by the function $P(t) = 3.5(1.005)^t$, where t is the number of years since 1960.

   (a) Compute $P(30)$.

   $P(30) = 4.065$ billion [1]

   (b) Find the derivative function $P'(t)$ of $P(t)$.

   $P'(t) = 3.5(1.005)^t \ln(1.005)$ [3]

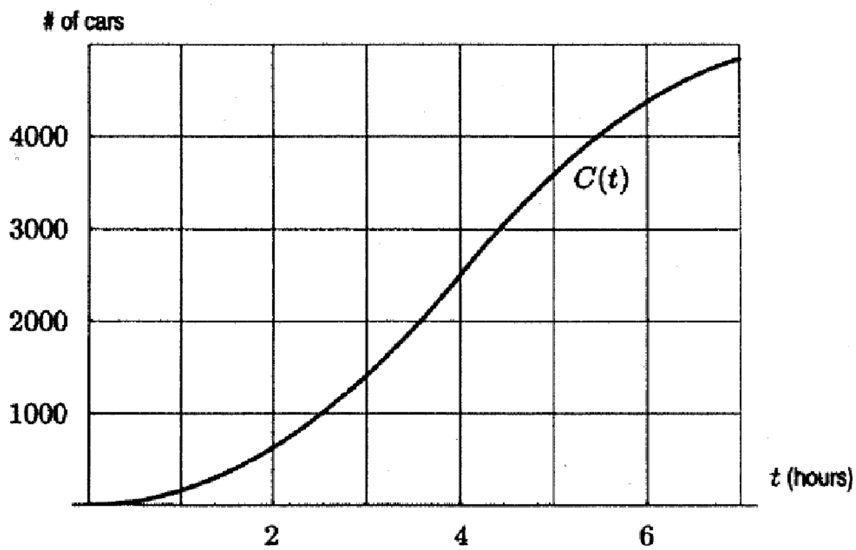
   (c) Compute $P'(30)$.

   $P'(30) = .0203$ billion/yr [1]

   (d) Explain what parts a and c mean. Be sure to include units in your explanation. [4]

   Solution.
   (a) The population of the world in 1990 was approximately 4.065 billion people.
   (c) In 1990, the population is growing by approximately .0203 billion people per year. □

8. To study the traffic flow along a major road, a city installs a device at the edge of the road at 4:00 a.m. The device counts the number of cars driving past, and records the total periodically. The resulting data is plotted on a graph, with time (in hours) on the horizontal axis and the number of cars on the vertical axis. The graph is shown below as the graph of a function $C(t)$.



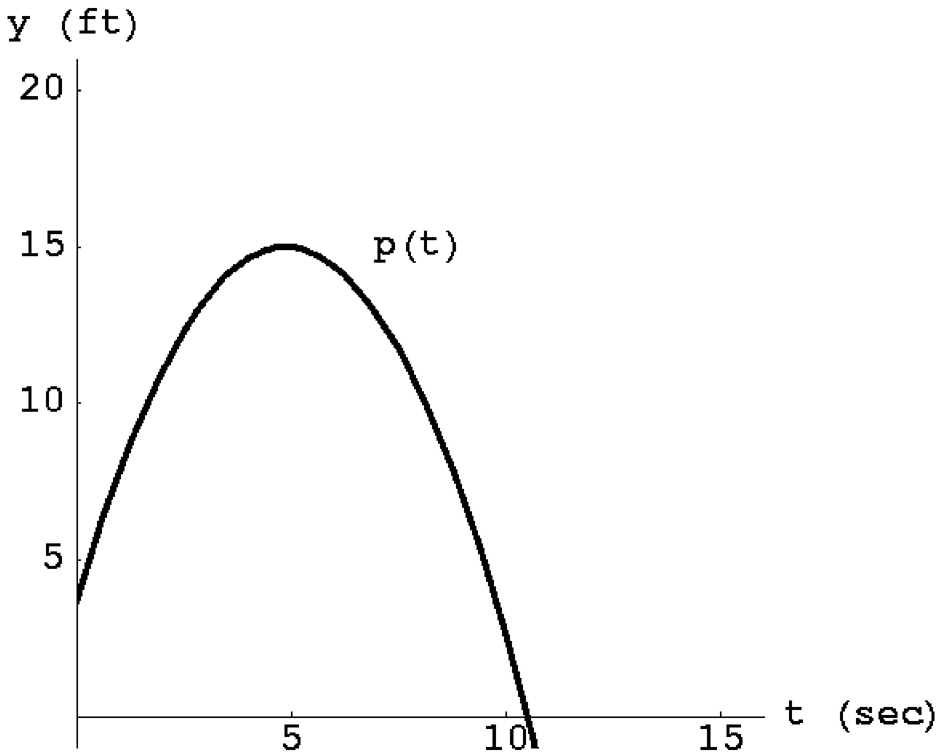
(a) From the graph, estimate $C'(3)$. $C'(3) \approx 1000$ cars/hr [3]

(b) What does the value of $C'(3)$ you obtained in (a) mean in pratical terms? What are the units? [4]

Solution.
At 7am, approximately 1000 cars drive past the device per hour.
C'(3) is measured in cars/hr. □

(c) Estimate when the traffic flow is greatest. 7:30 am [3]

9. The graph of $p(t)$ below gives the position of a particle at time t. List the following quantities in order, smallest to largest.



- A, average velocity on $1 \leq t \leq 3$
- B, average velocity on $8 \leq t \leq 10$
- C, instantaneous velocity at $t = 1$
- D, instantaneous velocity at $t = 3$
- E, instantaneous velocity at $t = 10$

EBDAC [5]

Name:______________________________

Non-Calculator section: You may NOT use your calculator on this section. You must show all work in order to recieve full credit.

1. Consider the function $y = 8e^{-2x} + 3$.

   (a) What happens to y as $x \to \infty$? $y \to 3$ [3]

   (b) What happens to y as $x \to -\infty$? $y \to \infty$ [3]

2. Find the equation of the tangent line to the graph of $g(x) = \frac{x^2-2}{x+1}$ at the point at which $x = 1$.

$y = \frac{5}{4}x - \frac{7}{4}$ [10]

3. Use the limit definition of the derivative to find the derivative of $f(x) = 2x^2 - 5x$.

$f'(t) = 4x - 5$ [8]

4. Find the derivatives of the following functions using any of the techniques you know.

(a) $y = 5x^4 - 3x^3 + 2x^2 - 8$

$\frac{dy}{dx} = 20x^3 - 9x^2 + 4x$ [3]

(b) $z = x^{-3}(3x^4 + 2)$

$\frac{dz}{dx} = 3 - 6x^{-4}$ [3]

5. Find the derivatives of the following functions using any of the techniques you know.

(a) $f(x) = \sqrt{x} - (\frac{1}{2})^x$

$$f'(x) = \tfrac{1}{2}x^{-1/2} - (\tfrac{1}{2})^x \ln(\tfrac{1}{2}) \quad [3]$$

(b) $g(x) = xe^x$

$$g'(x) = xe^x + e^x \quad [3]$$